LETTER TO THE EDITOR

Eikonal Approximation to the Low-Angle Light Scattering

Dear Sir:

In a paper published in this journal, Morris and Jennings (1) used anomalous diffraction approximation (1,2) to derive analytical solutions to low-angle light scattering from two homogeneous and isotropic concentric spheres of radii a_1 and a_2, with $a_1 > a_2$ and refractive indices n_1, n_2 embedded in a homogeneous and isotropic medium of refractive index n_0.[1] In this letter we wish to point out yet another approximation, closely related to anomalous diffraction approximation, known as eikonal approximation (3,4). In this approximation all the results of Ref. 1. retain the same form. The only difference between eikonal approximation (EA) and anomalous diffraction approximation (ADA) is that the phase shifts $\rho_{1(2)} = 2ka_{1(2)}(n_{1(2)} - n_{0(1)})/n_0$ in ADA are replaced by $\rho_{1(2)} = ka_{1(2)}(n_{1(2)}^2 - n_{0(1)}^2)/n_0^2$ in EA, k being the wave number of the incident ray.

The advantage of EA is that it is expected to be valid for arbitrary $\rho_{1(2)}$ values if the conditions $ka_{1(2)} \gg 1$ and $(n_{1(2)}^2 - n_{0(1)}^2)/n_0^2 \ll 1$ are satisfied, whereas ADA is expected to be good only if $\rho_{1(2)} \gg 1$. Hence, if either ρ_1 or ρ_2 or both ρ_1 and ρ_2 are of order unity it will be more appropriate to use respective EA phases rather than ADA phases. For the sake of clarity we recall (1,2) that analytical solutions are possible only when $\rho_{1(2)}$ is large ($\rho_{1(2)} > k\theta a_{1(2)}$) and when $\rho_{1(2)}$ is small ($\rho_{1(2)} < 1$), θ being the angle between incident and outgoing ray directions. Thus, in the present context, the region $\rho \approx 1$ also has to satisfy either one of the two conditions given above for the analytic solutions to be possible.

At first sight it may appear that if $n_0 \approx n_1 \approx n_2$, both EA and ADA will be equally good. However, to test this in the regions of physical interest, we have carried out a detailed numerical comparison of ADA, EA, and Rayleigh-Gans-Debye (RGD) approximation (5) with exact Mie theory (6) results for a homogeneous and isotropic sphere over a wide range of n and ρ values. Here, ρ is the phase shift suffered by the light rays in passing through a sphere of relative refractive index n and radius a. The detailed results will be published elsewhere. Here we list a few numbers in Table I to give an idea of the region

[1] For convenience, we use the notation n_0 instead of n used in Ref. 1.

TABLE I

COMPARISON OF EA, ADA, RGD, AND MIE THEORY RESULTS

$i = \|s(0)\|^2$	n	ka = 1.0	2.0	8.0	15.0
i_{Mie}	1.05	0.113×10^{-2}	0.741×10^{-1}	304.2	12.05×10^3
i_{EA}		0.117×10^{-2}	0.745×10^{-1}	294.0	11.54×10^3
i_{ADA}		0.111×10^{-2}	0.709×10^{-1}	280.6	11.05×10^3
i_{RGD}		0.109×10^{-2}	0.698×10^{-1}	286.1	12.40×10^3
i_{Mie}	1.20	0.189×10^{-1}	1.322	33.10×10^2	22.0×10^3
i_{EA}		0.212×10^{-1}	1.315	25.94×10^2	11.4×10^3
i_{ADA}		0.176×10^{-1}	1.095	24.69×10^2	16.6×10^3
i_{RGD}		0.163×10^{-1}	1.045	42.88×10^2	12.4×10^4

of validity of EA and the difference between it and ADA, RGD, and Mie theory results for some ρ values near unity. The quantity tabulated is $i = |s(0)|^2$, $s(0)$ being the amplitude function in the forward direction. Results clearly indicate the usefulness of EA in the region $\rho \approx 1$ and $(n^2 - 1) \ll 1$. As n deviates more from unity and ρ also increases, ADA proves to be a better approximation compared to EA (see the comparison for $n = 1.20$, $ka = 15$ in Table I).

Received for publication 7 October 1977.

REFERENCES

1. MORRIS, V. J., and B. R. JENNINGS. 1977. Anomalous diffraction approximation to the low-angle scattering from coated spheres. A model for biological cells. *Biophys. J.* **17**:95.
2. VAN DE HULST, H. C. 1957. Light scattering by small particles. John Wiley and Sons, Inc., New York. 181-184.
3. NEWTON, R. G. 1966. Scattering theory of waves and particles. McGraw-Hill Book Company, Inc., New York. 582-584.
4. SOMMERFELD, A. 1954. Lectures in theoretical physics. Vol. IV. Academic Press, Inc., New York. 207-210.
5. VAN DE HULST, H. C. 1957. Light scattering by small particles. John Wiley and Sons, Inc., New York. 85-87.
6. VAN DE HULST, H. C. 1957. Light scattering by small particles. John Wiley and Sons, Inc., New York. Chap. 9.

S. K. SHARMA AND S. DEBI
Saha Institute of Nuclear Physics
Calcutta-700 009 India